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14MAR/MAU/IAE/MDE/MMD/MST/MTH/MTE/MTP/
MTR/MCM/MEA/CAE11

First Semester M.Tech. Degree Examination, June/July 2016
Applied Mathematics

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to four significant digits and find its absolute and relative errors. (07 Marks)
- b. A parachutist of mass 68.1 kg jumps out a stationary hot air balloon. Use equation $v(t) = \frac{gm}{C} \left[1 - e^{-\left(\frac{C}{m}\right)t} \right]$ to compute velocity prior to opening the chutes. The drag coefficient is equal to 12.5 kg/s. (07 Marks)
- c. The Maclaurin's expansion for e^x is given by $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} e^r$, find 'n' such that the sum yields the value of e^x correct to 8 decimal places at $x = 1$. (06 Marks)

- 2 a. Find a root of the equation $xe^x = \cos x$ correct to four decimal places by Regula – False method. (10 Marks)
- b. Discuss Newton – Raphson method with the graph to find the root of the equation $f(x) = 0$. Use this method to find the root of the equation $X \log_{10}^x - 1.2 = 0$. Take the initial of x as 2. (10 Marks)

- 3 a. Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$ using initial approximation $p = 0.5, q = 0.5$. (10 Marks)
- b. Find all the roots of the polynomial $x^4 - x^3 + 3x^2 + x - 4 = 0$ using Graeffe's root squaring method, squaring thrice. (10 Marks)

- 4 a. Calculate first and second derivatives of the function tabulated in the following table at the point $x = 2.2$ and also find $\frac{dy}{dx}$ at $x = 2.0$. (10 Marks)

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7083	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

- b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method correct to four decimal places with $h = 0.5, h = 0.25$ and $h = 0.125$. (10 Marks)
- 5 a. Solve the following system of equations by the Gauss – Jordan method :
 $x_1 + x_2 + x_3 + x_4 = 2$
 $2x_1 - x_2 + 2x_3 - x_4 = -5$
 $3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$
 $x_1 - 2x_2 - 3x_3 + 2x_4 = 5$. (10 Marks)

- b. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, using partition method. (10 Marks)

- 6 a. Using Jacobi method find all the eigen values and the corresponding eigen vectors of the

$$\text{matrix } A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$

(10 Marks)

- b. Reduce the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ into tridiagonal form using householder's transformation and hence find its all eigen values.

(10 Marks)

- 7 a. Define linear transformation, onto and one – one transformation. IF $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation with A as the standard matrix for T, then prove that :

- i) T maps \mathbb{R}^n onto \mathbb{R}^m if and only if columns of A span \mathbb{R}^m
 ii) T is one – one if and only if columns of A are linearly independent.

(10 Marks)

- b. Given $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$ $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and define the transformation

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ by } T(x) = AX$$

- i) Find $T(u)$
 ii) Find an X in \mathbb{R}^2 , $T(X) = b$
 iii) Is there more than one X , whose image under T is b?
 iv) Determine if C is the range of transformation T.

(10 Marks)

- 8 a. Explain the Gram–Schmidt process of obtaining orthogonal basis for given basis $\{x_1, x_2, \dots, x_p\}$ for subspace W of \mathbb{R}^n and hence obtain orthonormal basis for $\{x_1, x_2, x_3\}$

$$\text{where } X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

(10 Marks)

- b. Find a least square solution of the inconsistent system $AX = b$ $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ and also

determine least square error.

(10 Marks)
